

(26)

Pliny's wife's chance of winning the total $(a+b)$ with a possession of x sesterces is her.

At every trial, she may win or lose 1 sesterce with a $P = \frac{1}{2}$

$$P(x) = \frac{P(x+1)}{2} + \frac{P(x-1)}{2}, \quad x > 0$$

$$\Rightarrow 2P(x) = P(x+1) + P(x-1)$$

$$\Rightarrow P(x) - P(x-1) = P(x+1) - P(x)$$

$$\Rightarrow P(x+1) - P(x) = P(x) - P(x-1) \quad \text{recursive formula}$$

$$= P(x-1) - P(x-2)$$

$$= P(1) - P(0)$$

$$= P(1) \quad (\because P(0) = 0)$$

$$P(2) = 2P(1)$$

$$P(3) = 3P(1) \Rightarrow P(n) = nP(1)$$

$$P(a+b) = 1, \text{ so } P(1) = \frac{1}{a+b}$$

$$\text{From this } P(a) = aP(1) = \frac{a}{a+b}$$